

Lecture 4: Splitting and killing

ACM 206, April 25, 2023

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4. Travel for a long time (e.g., length 100) without using the same vertex twice.

Self-avoiding walks

A Monte Carlo approach

1. Simulate a random walk.
2. Return to origin on self-intersection.
3. Repeat until reaching the desired length.

Self-avoiding walks

A faster Monte Carlo approach

1. Start at the origin, randomly select from the available vertices.
2. Repeat until failure or the desired length is reached.
3. Compute the importance weight

$$w_T = \prod_{t=0}^{T-1} n_t, \quad n_t = \# \text{open vertices}$$

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